ferent positions of the mass center and the elastic axis. Figure 4 shows that the flutter speed increases as the mass center moves forward, and Fig. 5 shows that the flutter speed does not change much as the position of the elastic axis changes from 45 to 25% chord measured from the leading edge.

In general, Figs. 4 and 5 demonstrate that the time-integration and harmonic analysis methods agree quite well. The agreement becomes less at higher Mach numbers. This difference stems from the lack of agreement in unsteady coefficients in Tables 1 and 2. All the curves show a dip in the neighborhood of the design Mach number.

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Prediction of Range and Endurance of Jet Aircraft at Constant Altitude

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Nomenclature

a_i = coefficients defined in Eqs. (13); $i = -3, -1, 1$
a,b = constants defined in Eqs. (9)
C = thrust specific fuel consumption
C_0, C_1 = zeroth- and first-order thrust coefficients of fuel consumption
C_D, C_L = total drag and lift coefficients
C_{D_0} = parasite-drag coefficient
D = total aerodynamic drag force
E = endurance
K = induced-drag factor
L = aerodynamic lift force
q = dynamic pressure
R, R_s = cruising range, specific range
$(R_s)_{\max}$ = maximum value of specific range
S = wing area
T, t = thrust force and time
V, V_{md} = true airspeed at cruise and for minimum drag
V_{opt} = true airspeed for maximum range
$W, \dot{W}$ = aircraft gross weight and its time derivative
W_f, W_i = final and initial values of W
ρ = atmospheric density at cruising altitude

Introduction

WITH the cost of petroleum-based fuels continuing to spiral upward, the subject of range and endurance vs fuel economy has taken on renewed importance. Thus, the precise manner in which range and endurance are predicted is becoming more crucial. Perhaps the first published analyses of range for turbojet-powered aircraft (there were undoubtedly numerous early analyses of an unpublished nature, such as company reports, etc.) were those of Page[1] and Jonas,[2] both in 1947. However, Page was concerned with only specific range rather than total. Jonas derived the square-root range expression (see Appendix A). In deriving this equation, he assumed that angle of attack is maintained at that required for maximum range and that engine speed and airspeed are decreased as fuel is consumed in the course of the flight. The same form of equation was derived in the well-known textbooks of Perkins and Hage[3] and Dommasch et al.[4]

Reference 2 was followed by the work of Ashkenas,[5] who obtained a similar expression for jet-aircraft range at altitudes below 35,000 ft and a logarithmic form (see Appendix B)[6-12] for the constant-temperature stratosphere.

It is shown here that neither the square-root (Appendix A) nor the logarithmic (Appendix B) form are appropriate for constant-altitude cruise because of the following.

1) The assumption that the thrust specific fuel consumption (C) is constant, regardless of thrust level, is a poor representation of actual turbojet or turbofan characteristics.

2) At constant altitude, the sets of variables (ρ, C, $C_L^{1/2}/C_D$) or (C, L/D, V) cannot be held constant when W is changing. However, both Ashkenas[5] and Nicolai[10] noted that the latter set can be held constant in a cruise climb by holding

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throttle setting (and thus C) and angle of attack (and thus L/D) constant and gaining altitude. However, this is not practical in today's heavy air traffic, for even military aircraft must conform to FAA-assigned constant flight altitudes.

The analysis presented here removes these two deficiencies and obtains a simple, closed-form expression which is more appropriate for constant-altitude cruise then either of the standard expressions (Appendix A or B) and yet is much more convenient, especially for preliminary design, than step-by-step integration.

Basic Equations

The specific range (R_s) is defined as the increase in range (R) per unit decrease in gross weight (W) due to burning of fuel. Thus, mathematically,

$$R_s = -\frac{\mathrm{d}R}{\mathrm{d}W} \tag{1}$$

Then the total cruising range is

$$R = -\int_{W_i}^{W_f} R_s(W)\,\mathrm{d}W \tag{2}$$

where W_i and W_f are the respective gross weights at the beginning and end of the cruise phase. Since $\mathrm{d}R/\mathrm{d}t$ = velocity (V) and denoting $\mathrm{d}W/\mathrm{d}t$ by $\dot{W}$, one can rewrite Eq. (1) as

$$R_s = -V/\dot{W} \tag{3}$$

Assuming that the flight conditions approach equilibrium, we have

$$W=L; \quad T=D \tag{4}$$

The endurance or time of flight associated with given conditions is

$$E = \int \mathrm{d}t = -\int_{W_i}^{W_f} (1/\dot{W})\,\mathrm{d}W \tag{5}$$

Derivation of New Equations

The following linear relationship between weight change due to fuel consumption and thrust[13] is used:

$$-\dot{W} = C_0 + C_1 T \tag{6}$$

Here, C_0 and C_1 are constants for a given altitude and airspeed. The relationship is much more realistic than Eq. (A1), which was used in all previous turbojet range and endurance analyses. Generally, $\dot{W}$ is also a function of altitude and airspeed (V). However, here both altitude and V are held constant, so that Eq. (6) is adequate, provided that C_0 and C_1 are determined for the desired conditions.

As angle of attack is varied, the total drag coefficient is assumed to be given by the familiar symmetric polar relation consisting of the sum of the parasite and induced drag components

$$C_D = C_{D_0} + KC_L^2 \tag{7}$$

where C_{D_0} and K are constants for a given aircraft in cruising configuration (i.e., no flap deflection, etc.). The flight strategy used here is to hold altitude and airspeed constant as W changes due to fuel consumption. The quantities controlled (i.e., varied) to maintain altitude and V constant are angle of attack (and thus C_L and C_D) and throttle setting (T and thus $\dot{W}$).

Using Eqs. (A2), (A3), (3), (6), and (7) in Eq. (2), we obtain

$$R = -\int_{W_i}^{W_f} \frac{b\,\mathrm{d}W}{a^2+W^2} \tag{8}$$

Here, the constants a^2 and b are given by

$$a^2 \equiv [C_0 qS + C_1 C_{D_0}(qS)^2]/KC_1; \quad b \equiv VqS/KC_1 \tag{9}$$

and $q=\rho V^2/2$ is the dynamic pressure. Equation (8) is integrated to give

$$R = (b/a)\,[\arctan(W_i/a) - \arctan(W_f/a)] \tag{10}$$

Since V = const, the endurance is given simply by

$$E = R/V = (b/aV)\,[\arctan(W_i/a) - \arctan(W_f/a)] \tag{11}$$

Application to Performance Prediction

Consider a hypothetical light business jet aircraft on a long-range ferrying flight at cruising conditions (400 knots at 45,000 ft).

Performance conditions:

$V = 460.6$ mph or 675.6 ft/s; $\rho = 0.4623\times10^{-3}$ slug/ft^3

$q = 105.5$ lb/ft^2

Aircraft and engine parameters:

$C_{D_0} = 0.0154$; $K = 0.0471$; $S = 100$ ft^2; $W_i = 2830$ lb

$W_f = 1780$ lb; $C_0 = 22.0$ lb/h; $C_1 = 0.623$ lb/lb-h

Then $a = 6656$ lb, $b = 1.656\times10^8$ lb-mi and directly from Eq. (10), $R = 3501$ mi. Finally, $E = R/V = 7.601$ h.

The equations of Appendices A and B are much less convenient to apply, since C_L, C_D, T, W, and C are not known directly. However, for comparative purposes, we will carry out these calculations. To apply Appendix A, assume that the effective gross weight on which to base C_L, C_D, T, etc. is simply the arithmetic mean value, i.e., 2305 lb. Then

$$C_L = 2305/(105.5\times100) = 0.2185$$

$$C_D = 0.0154 + (0.0471)(0.2185)^2 = 0.01765$$

$$T = D = (0.01765)(100)(105.5) = 186.2 \text{ lb}$$

$$\dot{W} = -[22.0 + 0.623(186.2)] = -138.0 \text{ lb/h}$$

$$C = -\dot{W}/T = 138.0/186.2 = 0.7412 \text{ lb/lb-h}$$

Using Eq. (A2), we get $R = 3528$ mi, only 0.77% higher than the exact value. We obtain 7.745 h from Eq. (A3). Note that 3528/7.745 = 455.5 mph, which is lower than the initial speed of 460.6 mph. Thus, we conclude that Eq. (A3) gives a reasonable result.

To apply Appendix B, we again use $W = 2305$ lb as the basis. Then from Eq. (B1), we obtain 3567 mi, and from Eq. (A3), again we get 7.745 h. Even in this long-range example, the Eq. (B1) result is only 1.89% higher than the exact result of Eq. (10).

Application to Determining Optimum Airspeed for Maximum Range

Equation (8) can be rewritten in terms of specific range (R_s) as

$$R_s^{-1} = a_{-3}V^{-3} + a_{-1}V^{-1} + a_1 V \tag{12}$$

For the aircraft mentioned in the previous example

$$a_{-3} \equiv 2KC_1W^2/\rho S = \frac{2\times0.0471\times0.623\times(2305)^2}{0.0004623\times100\times3600}$$

$$= 1874 \text{ lb-ft}^2/\text{s}^3$$

$$a_{-1} \equiv C_0 = 22.0/3600 = 6.111 \times 10^{-3}\ \text{lb/s}$$

$$a_1 \equiv \tfrac{1}{2} C_1 C_{D_0} \rho S = \frac{(\tfrac{1}{2}) \times 0.623 \times 0.0154 \times 0.0004623}{3600}$$

$$= 0.6160 \times 10^{-8}\ \text{lb-s/ft}^2 \tag{13}$$

Due to the relatively small value of a_1, it can be shown that R_s increases almost indefinitely with increasing V. However, use of Eqs. (4), (7), and (A1) results in the following expression for the maximum speed achievable with the thrust that is available:

$$V_{\max} = (1/\rho S)\{(T/C_{D_0}) + [(T/C_{D_0})^2 - (4KW^2/C_{D_0})]^{1/2}\} \tag{14}$$

Here, available $T = 234$ lb. This gives $V_{\max} = 779$ ft/s and Eq. (12) gives $R_s = 81{,}370$ ft/lb or 100.2 mi/gal (fuel weight = 6.5 lb/gal).

The airspeed (V_{md}) at which the drag is a minimum is given by [10,14]

$$V_{\text{md}} = (K/C_{D_0})^{1/4}(2W/\rho S)^{1/2} = 417.5\ \text{ft/s} \tag{15}$$

Thus, $V_{\text{opt}}/V_{\text{md}} = 779/418 = 1.864$, in contrast to $V_{\text{opt}}/V_{\text{md}} = (3)^{1/4} = 1.316$ obtained from classical theory (Appendix B type) by Hale and Steiger.[14] The present theory yields a higher optimum airspeed because the C predicted by Eq. (6) is actually lower at higher thrust levels, rather than constant as assumed in Appendix B. For example, at 45,000-ft altitude, when T is increased from 100 to 200 lb, C decreases from 0.843 to 0.733 lb/lb-h.

The optimum airspeed obtained here is 15.3% more than the aforementioned design cruise speed (400 knots). However, the design specifications were for 400 knots, suggesting that the wing area (S) may be disproportionately large. However, S was sized by the maximum permissible stall speed for single-engine aircraft, engine performance limited use above 45,000 ft, and structural limitations prevented use of a higher effective aspect ratio than 8.45 (with an Oswald efficiency factor of 0.8). A similar conclusion was recently reached by Holmes.[15]

Concluding Remarks

A closed-form equation for range at constant altitude was derived and applied to an example problem. It was shown in this example that the logarithmic-form equation given in Refs. 5-12 gave a range less than 2% higher than the accurate one, while the square-root-form equation (Refs. 2-4) gave a range only about 0.8% higher than the accurate one. The conventional endurance equations gave a value less than 2% higher.

Appendix A: Derivation 1

In Refs. 2-4, it was assumed that

$$\dot{W} = -CT; \quad V = (2W/\rho S C_L)^{1/2}; \quad T = (C_D/C_L)W \tag{A1}$$

where C is a constant.

Combining Eqs. (2), (3), and (A1) and assuming that altitude (and thus ρ), C, and $\sqrt{C_L}/C_D$ all remain constant (a physical impossibility), one obtains

$$R = -\sqrt{2}\int_{W_i}^{W_f} \frac{\sqrt{C_L}/C_D}{C\sqrt{\rho S}} \frac{\mathrm{d}W}{\sqrt{W}} = \frac{2\sqrt{2}}{C\sqrt{\rho S}} \frac{\sqrt{C_L}}{C_D}(\sqrt{W_i} - \sqrt{W_f}) \tag{A2}$$

Combining Eqs. (5) and (A1), one obtains

$$E = -\int_{W_i}^{W_f} \frac{1}{C} \frac{C_L}{C_D} \frac{\mathrm{d}W}{W} = \frac{1}{C} \frac{C_L}{C_D} \ln \frac{W_i}{W_f} \tag{A3}$$

Appendix B: Derivation 2

Substituting Eqs. (3), (4), and (A1) into Eq. (2) and assuming that C, L/D, and V all remain constant (a physical impossibility at constant altitude when W is changing), one obtains

$$R = -\int_{W_i}^{W_f} \frac{V}{C} \frac{L}{D} \frac{\mathrm{d}W}{W} = \frac{V}{C} \frac{L}{D} \ln \frac{W_i}{W_f} = \frac{V}{C} \frac{C_L}{C_D} \ln \frac{W_i}{W_f} \tag{B1}$$

Since V = const, the endurance is also given by Eq. (A3).

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RALS/VCE Turbine Inlet Temperature and Engine Complexity Optimization Study

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Background

STUDIES conducted by General Electric in the 1975-1979 time period had identified the variable cycle engine and remote augmented lift system (VCE/RALS) as a very at-

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